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**Simulation and Risk Analytics: Homework 1**

# Monte Carlo Simulation as a device to verify the properties of a model or a theory

Orange Team 6:

*Team Lead:* Phillip **Domschke**

*Other Contributors:* Marc Zimmerman, Steve Neola, Wes Ledebuhr, Jacob Frost

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# EXECUTIVE SUMMARY

In this report, we will show through Monte Carlo simulations that the parameter estimates of linear regression models will follow normal distribution following the central limit theorem and the law of big numbers. We will also show that heteroscedasticity among the error will not affect the normality assumption, however it will affect the associated t-test in some degree. Further, we demonstrate that strong correlation between two variables will heavily influence the rejection rate of our parameter estimates to unacceptable levels of more than 99%.

We will break the individual tests down into five distinct parts A, B, C, D, and E.

# ASSUMPTIONS

## Model Assumptions

True regression equation, which will be verified through simulation:

Where,

# ANALYSIS

## Part A – Is the distribution of the betas the one suggested in theory?

In order to answer this and the other question we rely on Monte Carlo simulations, in which we simulate sets of 100 random observations, which follow the above described distributions. To get reliable results, we repeat the process 20,000 times and evaluate the aggregate results.

AS we can see in the results table below, all measures indicate that the parameter estimates follow a normal distribution. The mean and median are almost equal and very close to the true parameter. Furthermore, excess kurtosis and Skewness are approximately zero, which is another indicator for a normal distribution of the parameter estimates. Finally, the Anderson-Darling test verifies these indicators. In the Appendix are additional QQ plots as well as parameter distribution histograms to further illustrate the validity of these results.

Table 1

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Variable | True Parameter | Mean | Median | Variance | Excess Kurtosis | Skewness | Anderson-Darling | |
| x1 | 0.21 | 0.20936 | 0.21133 | 0.12273 | -0.0214 | -0.0027 | 0.29346 | p>0.25 |
| x2 | -0.9 | 0.89775 | 0.89649 | 0.05363 | 0.10742 | -0.0067 | 1.87194 | p>0.089 |
| x3 | 3.45 | 3.44804 | 3.44661 | 0.06948 | 0.13759 | -0.0045 | 0.69287 | p>0.074 |

## Part B – How many times is incorrectly rejected?

Using a traditional alpha level of for the t-test, we find that 1038 records are misclassified and rejected. That equates to approximately 5.19% of all parameter estimates. This outcome was completely expected considering the alpha level controls the risk of committing a type I error Therefore, we can expect the test to falsely reject the true parameter at the same level as alpha.

## Part C – Change in Variance and Introduction of Heteroscedasticity

In part C we show the effect of the violation of the OLS assumption of independent errors. In order to simulate this effect we change the variance of the error distribution to:

Table 2

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Variable | True Parameter | Mean | | Median | Variance | Excess Kurtosis | Skewness | Anderson-Darling | |
| x1 | 0.21 | | 0.20874 | 0.21076 | 0.12241 | 0.00864 | -0.0047 | 0.26671 | p>0.25 |
| x2 | -0.9 | | -0.8977 | -0.896 | 0.07181 | 0.04452 | -0.0053 | 0.4815 | p>0.237 |
| x3 | 3.45 | | 3.44758 | 3.44685 | 0.06929 | 0.17352 | 0.00518 | 0.88809 | p>0.024 |

As we can see from Table 2 all the assumption regarding normality still hold as well as the unbiased estimate of the true population parameter. However, we cannot trust the t-test anymore. This becomes evident in the fact that now 1879 out of all 20,000 simulations falsely reject the null hypothesis. This amounts to almost double the set alpha level of 5% with approximately 9.4%, respectively.

## Part D – Omitting X3 to Explore Possible Bias

In order to explore the effects of completely leaving out one of the explanatory variables in the parameter estimation process, we ran another set of 20,000 simulations with 100 observations each without this time. The results are very similar to the one we found in Part C. Table 3 illustrates that the normality and unbiased parameter estimates are still valid assumptions. As before, we cannot trust the results of our t-test, since it falsely rejects the null hypothesis about 9.55% of the time, when it truly should commit this mistake only 5% of the time. This amounts to 1909 misclassifications.

Table 3

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Variable | True Parameter | Mean | Median | Variance | Excess Kurtosis | Skewness | Anderson-Darling | |
| x1 | 0.21 | 0.20955 | 0.21107 | 0.12258 | 0.04844 | -0.0058 | 0.43387 | p>0.25 |
| x2 | -0.9 | -0.8992 | -0.9008 | 0.071 | 0.02756 | 0.00981 | 0.44047 | p>0.25 |

## Part E – Introducing Correlation between Variables

To show the effect of high correlation between two variables, we simulated the case in which have a correlation of 0.6.

# CONCLUSION

# APPENDIX











